

UNBIASED GLUON JET MULTIPLICITY FROM e^+e^- THREE-JET EVENTS

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The charged particle multiplicities of two- and three-jet events from the reaction $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ are measured for Z^0 decays to light quark (uds) flavors, using the data sample of the OPAL Collaboration at LEP. Using recent theoretical expressions to account for biases from event selection, results corresponding to unbiased gluon jets are extracted. The unbiased gluon jet data are compared to corresponding results for quark jets. We determine the ratio $r \equiv N_g/N_q$ of multiplicities between gluon and quark jets as a function of energy scale. We also determine the ratio of slopes, $r^{(1)} \equiv (dN_g/dy)/(dN_q/dy)$, and of curvatures, $r^{(2)} \equiv (d^2N_g/dy^2)/(d^2N_q/dy^2)$, where y specifies the energy scale. At 30 GeV, we find $r = 1.422 \pm 0.051$, $r^{(1)} = 1.761 \pm 0.071$ and $r^{(2)} = 1.98 \pm 0.13$, where the uncertainties are the statistical and systematic terms added in quadrature. These results are in general agreement with theoretical predictions.

1 Introduction

The mean charged particle multiplicity of a gluon jet has often been measured in the annihilation of an electron and positron to hadrons, $e^+e^- \rightarrow \text{hadrons}$. The usual method is to select three-jet quark-antiquark-gluon $q\bar{q}g$ final states for which the events and individual jets are defined using a jet algorithm. The particle multiplicity of a jet determined with this technique is found to depend on which algorithm is employed. Therefore, these jets and the associated $q\bar{q}g$ events are called “biased.” In contrast, theoretical calculations usually define gluon jet multiplicity inclusively, by the particles in hemispheres of gluon-gluon (gg) systems in an overall color singlet. Quark jets are defined analogously as hemispheres of quark-antiquark ($q\bar{q}$) systems. The hemisphere definition of jets yields results which are independent of a jet finder. Therefore, these jets are called “unbiased.” Unbiased gluon jet multiplicity has so far been measured only in Υ ^{1,2} and Z^0 ^{3,4,5} decays, corresponding to jet energies of about 5 and 40 GeV, respectively.

It is of interest to measure the multiplicity of unbiased gluon jets at other scales. Such measurements would allow a test of recent predictions ^{6,7} from Quantum Chromodynamics (QCD) for the scale dependence of the multiplicity ratio $r \equiv N_g/N_q$ between gluon and quark jets and for the related ratios of slopes $r^{(1)} \equiv (dN_g/dy)$ and of curvatures $r^{(2)} \equiv (d^2N_g/dy^2)$, where N_g and N_q are the mean particle multiplicities of gluon and quark jets, $y = \ln(Q/\Lambda)$,

Q is the jet energy and Λ is the QCD scale parameter. Recently, a method to extract the multiplicity of unbiased gluon jets from *biased* $e^+e^- \rightarrow q\bar{q}g$ events was proposed ^{8,9}, extending earlier formalism ¹⁰. By combining measurements of N_g found from this method with the unbiased measurements for N_q , the ratios r , $r^{(1)}$ and $r^{(2)}$ can be determined at a variety of scales and used to test the corresponding QCD results in a quantitative manner.

Note that the DELPHI Collaboration has also recently presented preliminary results testing the theoretical formalism of ^{8,9} (see ¹¹).

2 Analysis method

Analytic expressions for the mean particle multiplicity of e^+e^- three-jet events, valid to the next-to-leading order of perturbation theory (NLO, also called MLLA), were recently presented in ⁹:

$$N_{q\bar{q}g} = N_{q\bar{q}}(L, k_{\perp, Lu}) + \frac{1}{2} N_{gg}(k_{\perp, Lu}) \quad , \quad (1)$$

$$N_{q\bar{q}g} = N_{q\bar{q}}(L_{q\bar{q}}, k_{\perp, Lu}) + \frac{1}{2} N_{gg}(k_{\perp, Le}) \quad . \quad (2)$$

The reason for the two different expressions is that there is an ambiguity in the definition of the gluon jet transverse momentum when the gluon radiation is hard. $k_{\perp, Lu}$ and $k_{\perp, Le}$ are the transverse momenta of the gluon with respect to the quark-antiquark system using the definition of either the Lund ($k_{\perp, Lu}$) ⁸ or Leningrad ($k_{\perp, Le}$) ¹⁰ groups. $N_{q\bar{q}g}$ is the particle multiplicity of a three-jet event selected using a jet algorithm. $N_{q\bar{q}}$ and N_{gg} are the multiplicities of two-jet $q\bar{q}$ and gg systems, given by twice N_q and N_g , respectively. The scales L , $L_{q\bar{q}}$, $k_{\perp, Lu}$ and $k_{\perp, Le}$ are defined in ^{8,9}.

The multiplicity of the gg system in this formalism, N_{gg} , depends only on a single scale: $k_{\perp, Lu}$ in eq. (1) or $k_{\perp, Le}$ in eq. (2). This dependence on a single scale is a statement that N_{gg} is *unbiased*, i.e. $N_{gg}(k_{\perp})$ in eq. (1) or (2) is equivalent to the inclusive multiplicity of a gg event from a color singlet source produced at the same scale k_{\perp} , to NLO accuracy.

The theoretical formalism is based on massless quarks. Therefore, we select light quark (u, d and s) events for our study. Three-jet events are defined using a jet algorithm. For our standard analysis we employ the Durham jet finder. As a systematic check, we use the Cambridge and Luclus jet finders. The resolution scale of the Durham jet finder, y_{cut} , is adjusted separately for each tagged uds event so that exactly three jets are reconstructed. For the Cambridge jet finder, the resolution scale is again y_{cut} . For Luclus the corresponding parameter is d_{join} . The jets are ordered from 1 to 3 such that

jet 1 has the highest energy. The angle opposite jet 1 is called θ_1 , etc. Events are retained if the angles between the highest energy jet and the other two are the same to within 3° , the so-called “Y events”^{12,13}. For Y events, the three-jet event multiplicity and scales $L_{q\bar{q}}$, $k_{\perp,Lu}$ and $k_{\perp,Le}$, depend only on $E_{c.m.}$ and one inter-jet angle, conveniently chosen to be θ_1 . For $35^\circ \leq \theta_1 \leq 120^\circ$, the range of θ_1 we employ for our gluon jet analysis, 22 365 events are selected: this is our final event sample. For simplicity, we identify the gluon jet by assuming it is the lowest energy jet in an event, i.e. jet 3.

To find the $N_{q\bar{q}}(L, k_{\perp,Lu})$ terms in eq. (1), we employ two methods. First, for the standard analysis, we perform a direct measurement. Specifically we determine the particle multiplicity of two-jet uds events from Z^0 decays as a function of the jet resolution scale $k_{\perp,Lu}$. Second, as a systematic check, we evaluate the following analytic expression from⁸:

$$N_{q\bar{q}}(J, k_{\perp,Lu}) = N_{q\bar{q}}(J') + (J - J') \frac{d N_{q\bar{q}}(J')}{d J'} \quad , \quad (3)$$

where $J' = k_{\perp,Lu} + c_q$ with $c_q = 3/2$. The unbiased term $N_{q\bar{q}}(K)$ is equivalent to the mean multiplicity of inclusive $e^+e^- \rightarrow hadrons$ events as a function of $K = E_{c.m.}$. To find the $N_{q\bar{q}}(L_{q\bar{q}}, k_{\perp,Lu})$ terms in eq. (2), we utilize only the second of these methods, i.e. the one based on eq. (3), because a direct measurement is not straightforward in this case.

3 Results

The leftmost set of plots in Fig. 1 shows our results for the unbiased charged particle multiplicities of gg events, $N_{gg}^{ch.}$. The solid points in the top plot are obtained from eq. (1) using the direct measurements of $N_{q\bar{q}}^{ch.}(L, k_{\perp,Lu})$. The asterisks and open symbols in the bottom plot show the corresponding results from eqs. (1) and (2) using the calculated expressions for $N_{q\bar{q}}^{ch.}(J, k_{\perp,Lu})$ from eq. (3). The two sets of results from eq. (1) (solid points in the top left plot and asterisks in the bottom left plot) are seen to be very similar.

Included in the leftmost plots are direct measurements of unbiased gluon jet multiplicity from the CLEO^{1,2} and OPAL Collaborations⁵. Also shown are the predictions of the Herwig and Jetset Monte Carlo event generators for the inclusive charged particle multiplicity of gg events. It is seen that the Monte Carlo predictions describe the direct measurements of unbiased gluon jet multiplicity (triangle symbols) well. The results from the present analysis based on eq. (1) are also well described by the Monte Carlo curves. In contrast, the results from eq. (2) (open symbols in the bottom leftmost plot) are generally well above the Monte Carlo predictions, and – if extrapolated to

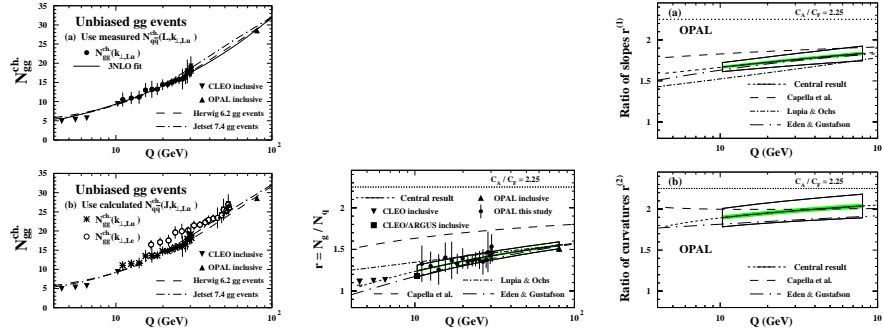


Figure 1. Leftmost two plots: the mean charged particle multiplicity of unbiased gg events as a function of scale. Central plot: the ratio of slopes r between unbiased gluon and uds quark jets. Rightmost two plots: the corresponding results for the ratios of slopes $r^{(1)}$ and of curvatures $r^{(2)}$.

lower and higher energies – appear inconsistent with the direct measurements from CLEO and OPAL as well. We therefore conclude that the equation based on the Lund definition of the gluon jet scale, eq. (1), yields results which are more consistent with other studies than the equation based on the Leningrad definition, eq. (2). We henceforth restrict our analysis of gluon jets to the former set of results.

We employ the following procedure to determine r , $r^{(1)}$ and $r^{(2)}$ from experiment. The corrected unbiased quark and gluon jet multiplicities are separately fitted using the next-to-next-to-next-to-leading order (3NLO) expressions for N_q and N_g ^{7,14}, respectively. To better constrain the results, the direct measurements of unbiased gluon jet multiplicity from CLEO and OPAL are included in the gluon jet fit. The ratio of the fitted expressions for N_g and N_q defines r . We calculate the first and second derivatives of the analytic equations for N_g and N_q with respect to y and evaluate the resulting expressions using the corresponding fitted parameter values. The ratios of these terms define $r^{(1)}$ and $r^{(2)}$.

Our results for r are shown in the central plot in Fig. 1, those for $r^{(1)}$ and $r^{(2)}$ in the two rightmost plots. The central results are indicated by solid curves. The shaded bands show the statistical uncertainties. The overall uncertainties, with statistical and systematic terms added in quadrature, are shown by the open bands. At 30 GeV, a typical scale in our analysis, we find $r = 1.422 \pm 0.006 \pm 0.051$, $r^{(1)} = 1.761 \pm 0.013 \pm 0.070$ and $r^{(2)} = 1.98 \pm 0.02 \pm 0.13$. This is consistent with the QCD prediction⁷ that $r < r^{(1)} < r^{(2)} < C_A/C_F =$

2.25 for the scales accessible in our study,

The 3NLO predictions of Capella et al. ⁷ for r , $r^{(1)}$ and $r^{(2)}$ are shown by the long-dashed curves in the central and rightmost plots of Fig. 1. The analytic predictions for r and $r^{(1)}$ exceed the corresponding experimental results by about 22% and 6%, while the theory agrees with the data for $r^{(2)}$. A second QCD prediction for r versus scale was recently presented by Lupia and Ochs ⁶. The predictions of this calculation for r and $r^{(1)}$ are shown by the short-dashed-dotted curves. At 30 GeV, r is predicted to be 1.45, in agreement with our measurement at that scale. The corresponding result for $r^{(1)}$ is 1.64, about 7% below the data. Finally, theoretical predictions for r , $r^{(1)}$ and $r^{(2)}$ can be derived from the formalism of Edén and Gustafson ⁸. These predictions are shown by the long-dash-dotted curves. The results are seen to be in good overall agreement with the data. We note, however, that these predictions are based on the experimental measurements of quark jet multiplicities. Therefore, the predictions we derive based on ⁸ are not entirely independent of the data.

In conclusion, we find overall agreement between the experimental and theoretical results.

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